EE.351: Spectrum Analysis and Discrete-Time Systems
MIDTERM EXAM, 2:30PM–4:30PM, November 4, 2004 **(closed book)**

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Note: **There are four questions. All questions are of equal value but not necessarily of equal difficulty. Part marks for each question are indicated. Full marks shall only be given to solutions that are properly explained and justified.**

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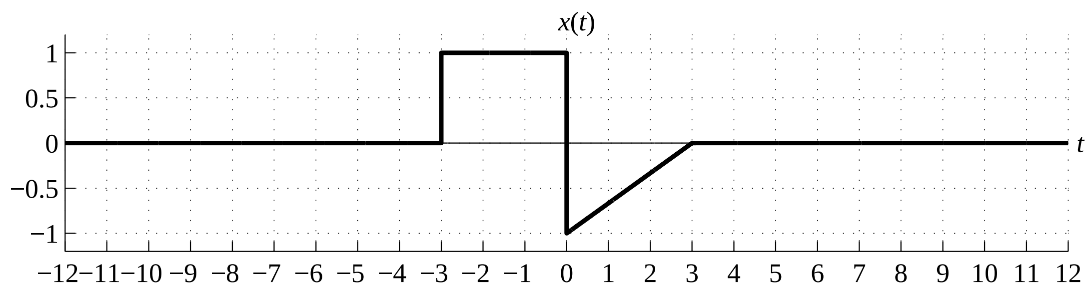
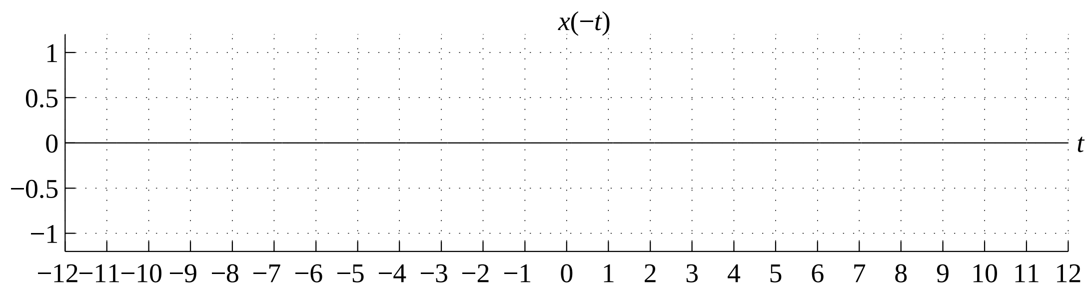
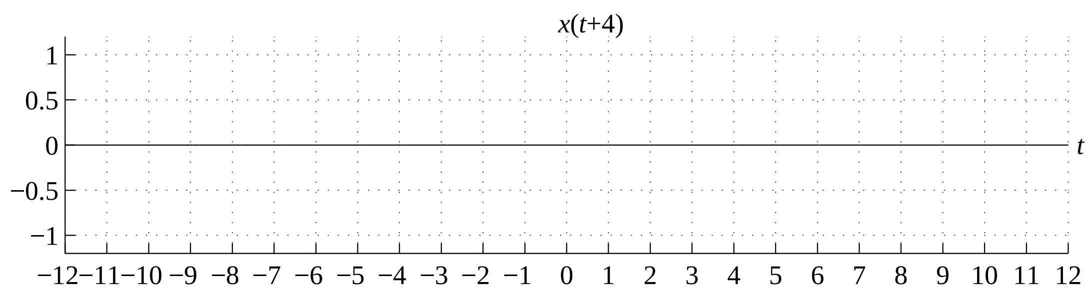
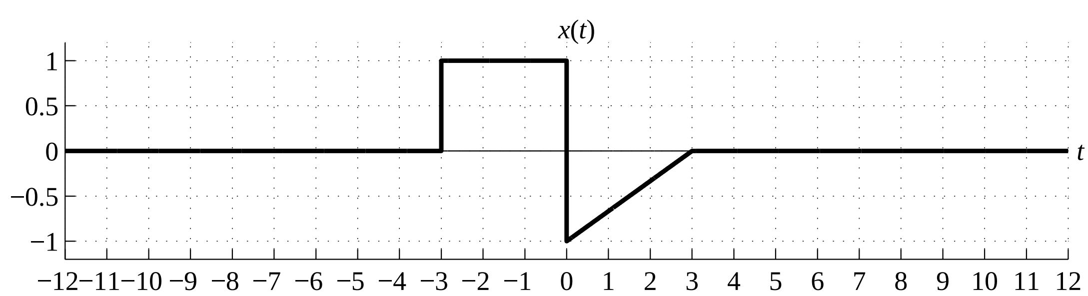
1. *(Signal Transformations)*

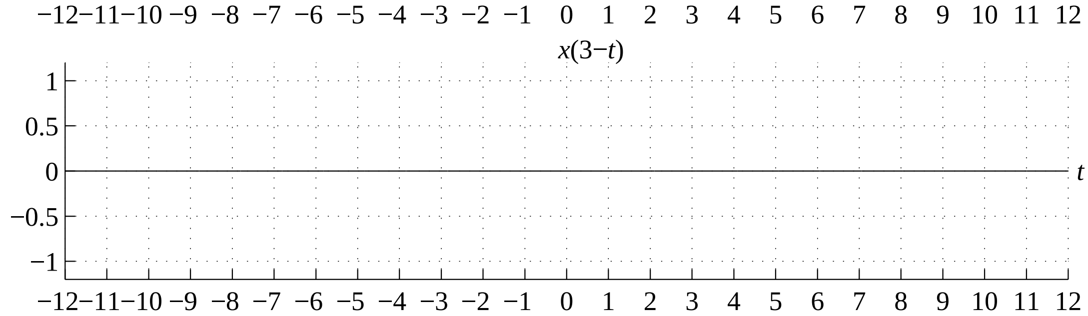
[6] (a) A continuous time signal $x(t)$ is shown below. Neatly sketch each of the following signals: (i) $x(-t)$, (ii) $x(t+4)$, (iii) $x(3-t)$, (iv) $x\left(-\frac{t}{2}\right)$.

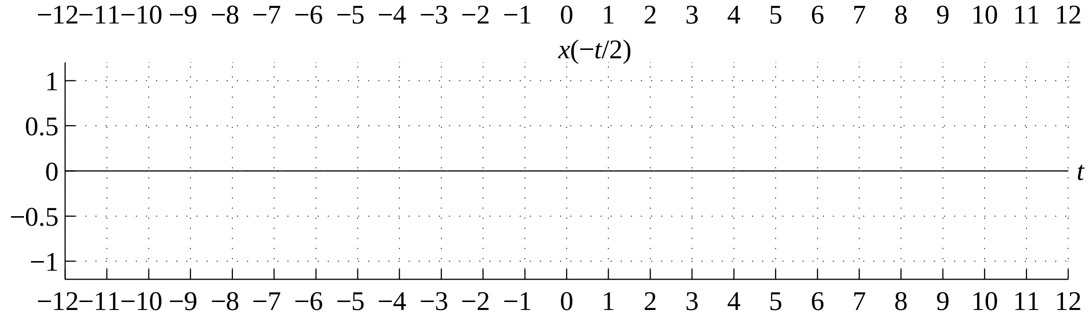




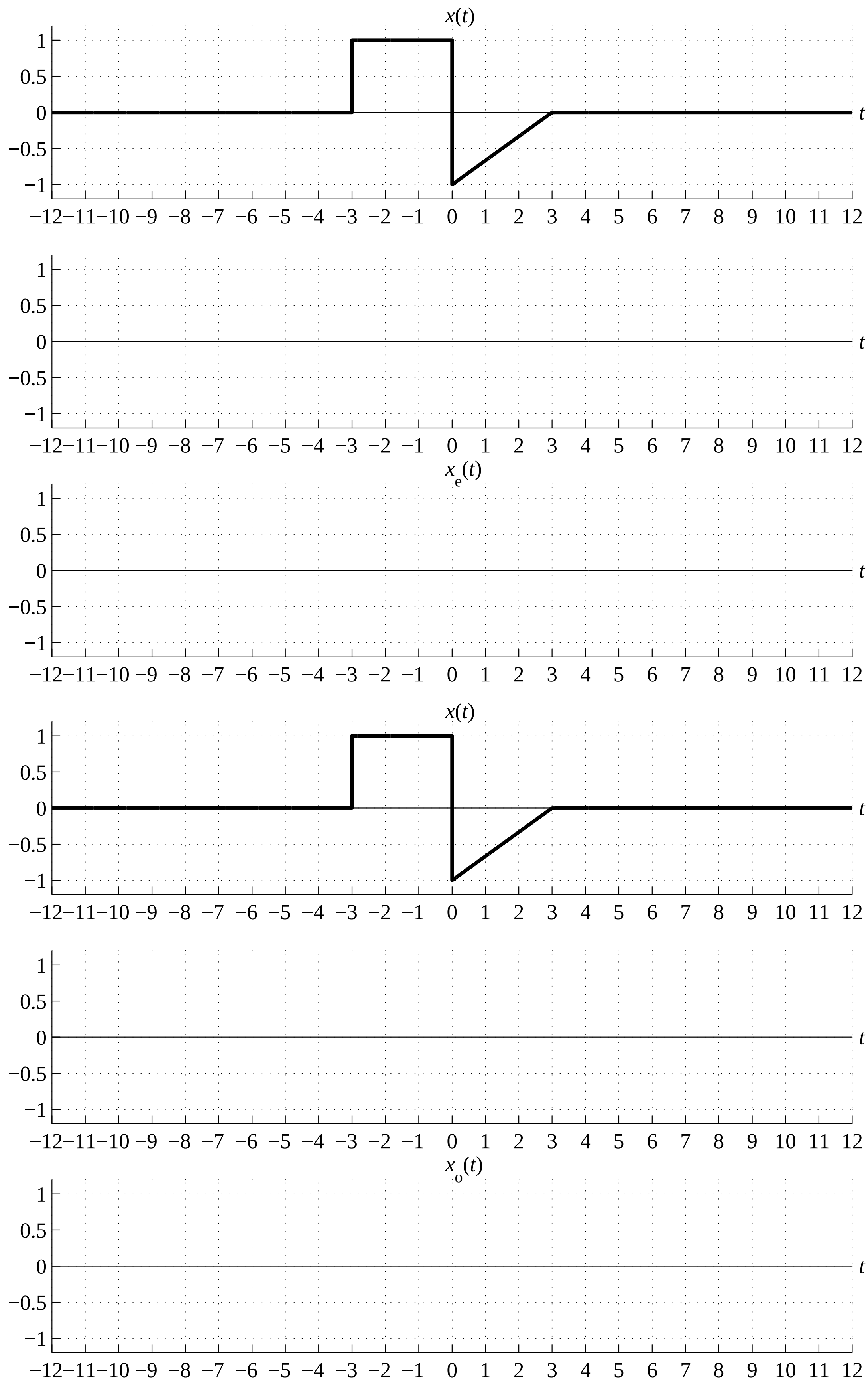








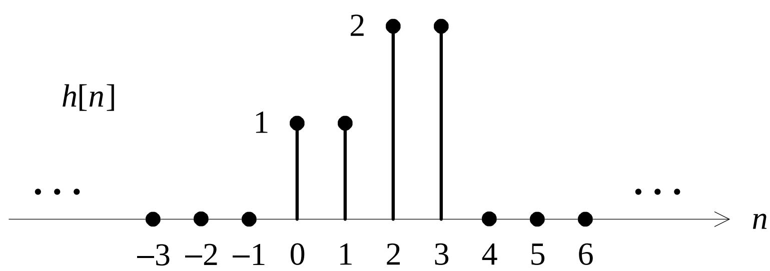
[3] (b) Consider the same continuous-time signal as in part (a). Determine and sketch its even and odd parts.
Note: The provided templates might be useful. You do not have to use all of them if you do not need to.

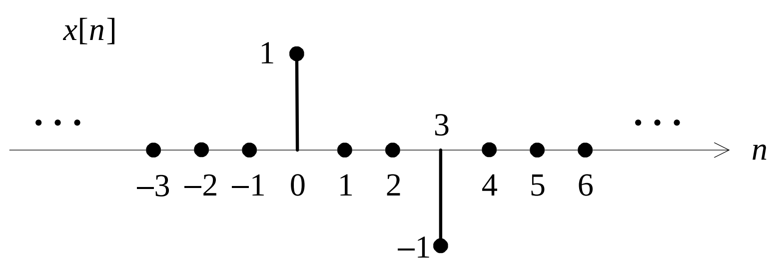


[1] (c) Is $x(t)$ an energy or power signal? Why?

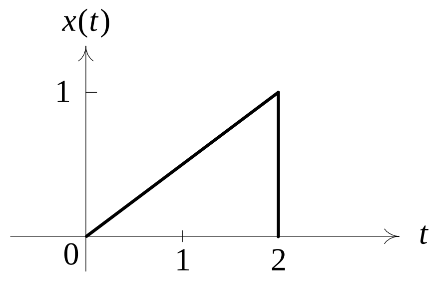
2. *(Convolution)*

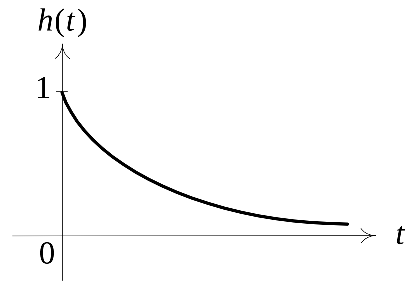
[5] (a) Consider a discrete-time LTI system with impulse response $h[n]$ and input $x[n]$ as shown below. Find and neatly sketch the output $y[n]$.
Hint: The output can be found by using the *Linearity* and *Time-Invariance* properties instead of performing the convolution sum.





[5] (b) Consider a continuous-time LTI system with impulse response $h(t) = \mathrm{e}^{-t}u(t)$ and input $x(t)$ as shown below. Find and roughly sketch the output $y(t)$.



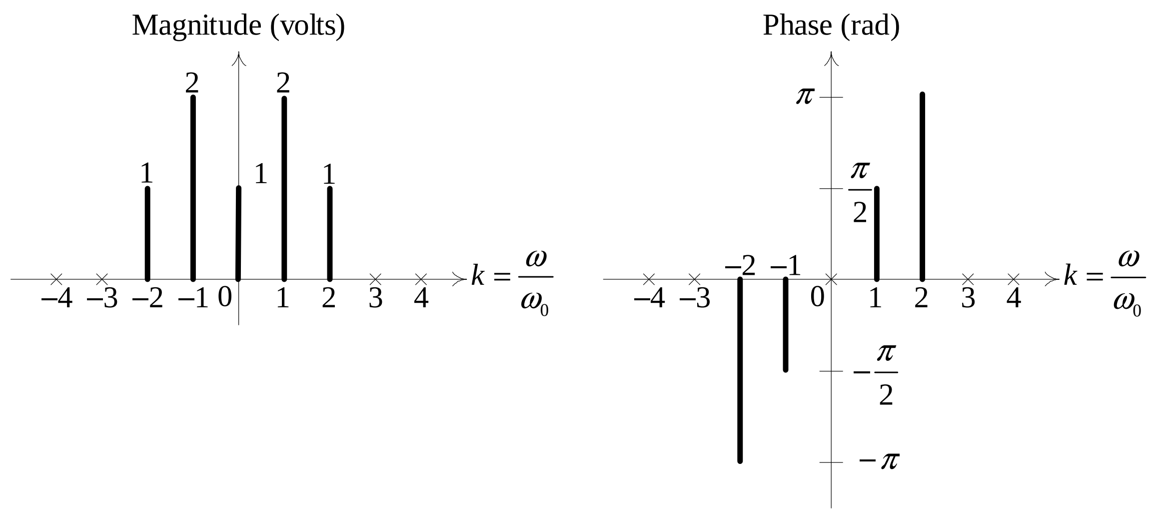


3. *(Properties of Fourier Series Coefficients)* The following figures show the magnitude and phase spectra of three continuous-time periodic signals. Answer the following questions:

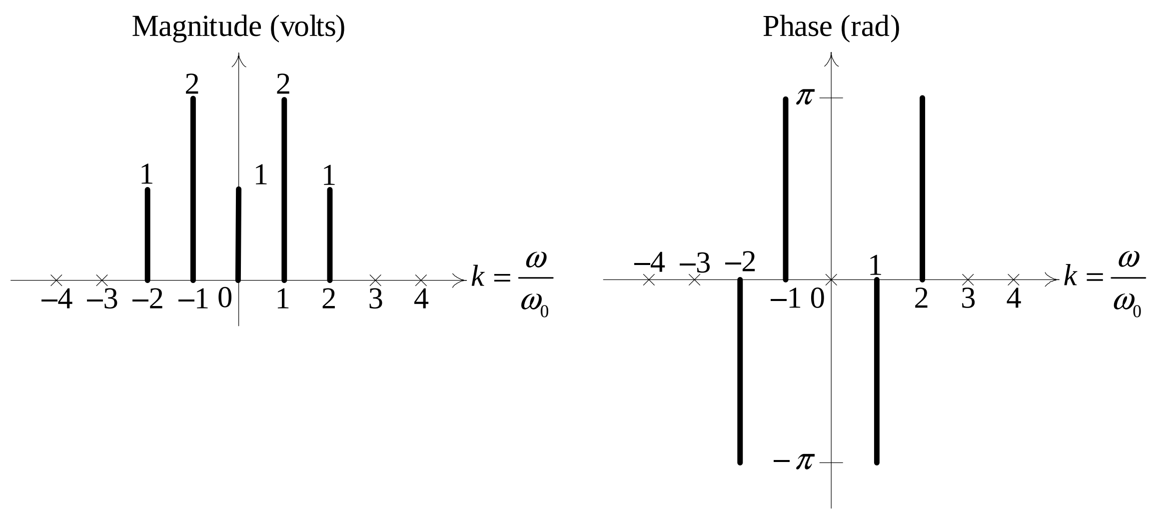
[4] (a) For each signal, determine whether the signal is real or complex. Explain your answer.

[2] (b) Which signal is a real-valued and even function? Which signal is a real-valued and odd function? Explain your answer.

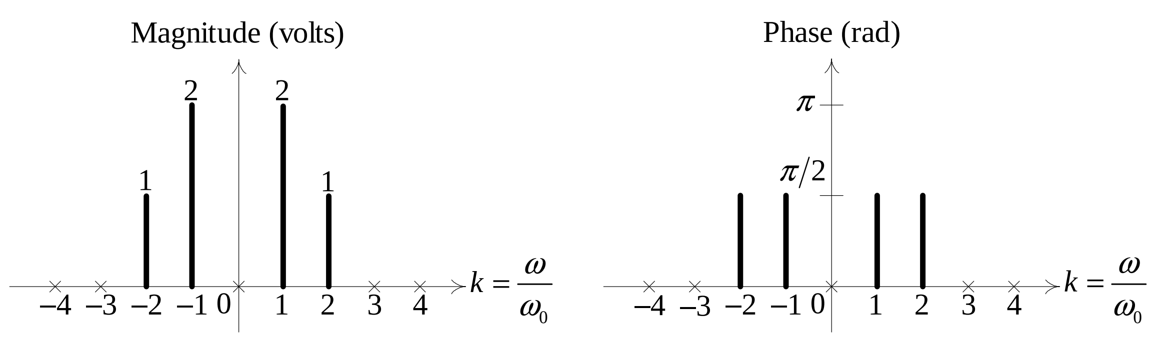
(i)



(ii)



(iii)



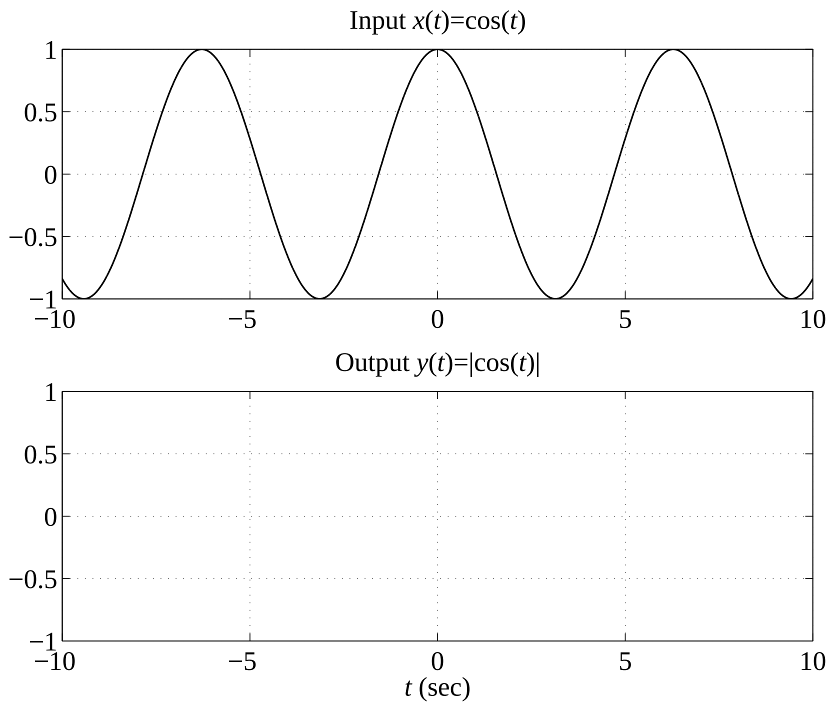
[4] (c) For the signal corresponding to the spectra in (i), do the following:
Write a Fourier series equation for the time waveform $x(t)$.

What is the dc component of this signal?

Find the average power of this signal.

4. *(Fourier Series Representation)* One technique for building a dc power supply is to take an ac signal and full-wave rectify it. That is, if the input to the full-wave rectifier is the ac signal $x(t)$, then its output is $y(t) = |x(t)|$.

[3] (a) Consider the input signal $x(t) = \cos(t)$ shown in the figure below. Neatly sketch the output $y(t)$. What are the fundamental periods of $x(t)$ and $y(t)$?



[5] (b) Find the trigonometric Fourier series coefficients B_k and C_k of the output $y(t)$.

[2] (c) What are the amplitudes of the dc components of the input and output signals, respectively?

Potentially Useful Facts:

- Even and odd parts:

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)], \quad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

- Convolution sum:

$$x[n] * h[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

- Convolution integral:

$$x(t) * h(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\mathrm{d}\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\mathrm{d}\tau$$

- FS representation of CT periodic signals (CTFS):

$$\begin{aligned}
\text{Exponential: } x(t) &= \sum_{k=-\infty}^{\infty} a_k \mathrm{e}^{jk\omega t} \\
\text{Amplitude-Phase: } x(t) &= a_0 + 2\sum_{k=1}^{\infty} A_k \cos(k\omega t + \theta_k) \\
\text{Trigonometric: } x(t) &= a_0 + 2\sum_{k=1}^{\infty} [B_k \cos(k\omega t) - C_k \sin(k\omega t)]
\end{aligned}$$

$$\begin{aligned}
a_k &= A_k \mathrm{e}^{j\theta_k} = B_k + jC_k \\
A_k &= |a_k| = \sqrt{B_k^2 + C_k^2} \qquad \theta_k = \angle a_k = \tan^{-1}\left(\frac{C_k}{B_k}\right) \\
B_k &= \Re\{a_k\} = A_k \cos(\theta_k) \qquad C_k = \Im\{a_k\} = A_k \sin(\theta_k) \\
a_k &= \frac{1}{T_0}\int_{T_0} x(t)\mathrm{e}^{-jk\omega t}\mathrm{d}t \\
B_k &= \frac{1}{T_0}\int_{T_0} x(t)\cos(k\omega t)\mathrm{d}t, \quad C_k = -\frac{1}{T_0}\int_{T_0} x(t)\sin(k\omega t)\mathrm{d}t
\end{aligned}$$

<u>*Remark*</u>: The amplitude-phase and trigonometric forms only apply for real-valued signals.

- The *complex-conjugate symmetry* of the Fourier series coefficients of real-valued periodic signals:

$$\boxed{a_{-k}^* = a_k, \qquad a_{-k} = a_k^*}$$

- *Parseval's relation*:

$$\boxed{\frac{1}{T_0}\int_{T_0} |x(t)|^2 \mathrm{d}t = \sum_{k=-\infty}^{\infty} |a_k|^2}$$

- Identities:

$$\begin{aligned}
e^{j\omega t} &= \cos(\omega t) + j\sin(\omega t) \\
e^{-j\omega t} &= \cos(\omega t) - j\sin(\omega t) \\
\cos(x)\cos(y) &= \frac{1}{2}[\cos(x+y) + \cos(x-y)]
\end{aligned}$$

- Indefinite Integrals:

$$\int t\mathrm{e}^{at}\mathrm{d}t = \frac{\mathrm{e}^{at}}{a^2}(at - 1)$$

$$\int \sin(bt)\mathrm{e}^{at}\mathrm{d}t = \frac{\mathrm{e}^{at}}{a^2 + b^2}[a\sin(bt) - b\cos(bt)]$$

$$\int \cos(bt)\mathrm{e}^{at}\mathrm{d}t = \frac{\mathrm{e}^{at}}{a^2 + b^2}[a\cos(bt) + b\sin(bt)]$$

$$\int \cos(at)\mathrm{d}t = \frac{1}{a}\sin(at)$$

$$\int \sin(at)\mathrm{d}t = -\frac{1}{a}\cos(at)$$